

## 1) Quiz Thursday

Back to the Heat Equation  
Two - Dimensional:  

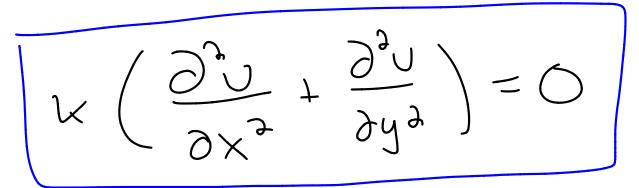
$$u = u(x, y, t)$$
 the  
temperature in a 2-D region  
 $k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \frac{\partial u}{\partial t}$ 

is the heat equation, where again, k is the thermal conductivity of the material.

line Independence

We assume that the value U(X,Y,t) does not depend on  $t = ) \frac{\partial U}{\partial t} = 0$ 

The time independent heat equation is



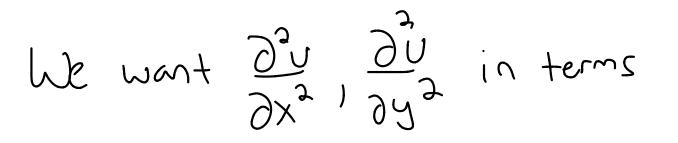
Circular Domains

We measure the heat 
$$flow$$
  
on a disk of radius  
 $R$ , so the region is  
 $D = \sum (r_{J} \Theta) | O \le r \le R, O \le \Theta < 2T \int_{a}^{a}$ 

Boundary conditions on 
$$U$$
 of  
the form  
 $U(R, \Theta) = f(\Theta)$   
(boundary circle of radius R)

Switch to Polar Coordinates

Recall we have  $\Gamma = \sqrt{\chi^2 + \gamma^2}$  $\Theta = \operatorname{arctan}\left(\frac{y}{x}\right)$ Can then write  $\upsilon = \upsilon(\Gamma, \theta)$  (t suppressed since u is time-independent). Rewrite the heat equation in polar coordinates!



of r and O. Use the Chain Rule!

 $\frac{\partial x}{\partial \theta} = \frac{\partial x}{\partial \theta} + \frac{\partial y}{\partial \theta} = \frac{\partial x}{\partial \theta} + \frac{\partial y}{\partial \theta} = \frac{\partial x}{\partial \theta}$  $= \frac{\partial U}{\partial r} \frac{\partial ((x^2 + y^a)^{V_a})}{\partial x} +$  $\frac{\partial U}{\partial A} = \frac{\partial U}{\partial X} \left( \arctan\left(\frac{y}{x}\right) \right)$  $= \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \left( -\frac{x^3}{2} \right) \left[ t \left( \frac{x}{2} \right)^3 \right]$  $= \frac{\partial U}{\partial x} \frac{X}{X^{2} + y^{2}} + \frac{\partial U}{\partial \theta} \left( \frac{-Y}{X^{2} + y^{2}} \right)$ 

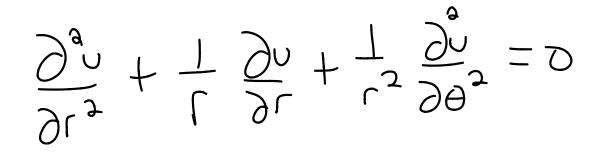
Similarly,

 $\frac{\partial v}{\partial y} = \frac{\partial v}{\partial r} \frac{\partial r}{\partial r} + \frac{\partial v}{\partial r} \frac{\partial \theta}{\partial \theta}$  $=\frac{\partial U}{\partial r}\frac{\partial}{\partial y}\left(\left(x^{2}+y^{3}\right)^{1/2}\right)+$  $\frac{\partial U}{\partial A} \frac{\partial}{\partial Y} \left( \operatorname{arctan}(\frac{Y}{X}) \right)$  $= \frac{\partial U}{\partial x^{2} + 2} + \frac{\partial U}{\partial y} \left(\frac{1}{x}\right) \left(\frac{1}{1 + (\frac{x}{2})^{2}}\right)$  $= \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Y}$ 

Extra Credit (10 points, due Tuesday after examd) Show that, in polar coordinates, the heat equation  $\frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial y^2} = 0$  becomes  $\frac{\partial L_{J}}{\partial J} + \frac{L}{J} \frac{\partial L}{\partial J} + \frac{L}{J} \frac{\partial D}{\partial J} = 0$ by using the chain rule to (ompute 200 + 200)

Example ) : Attempt to solve

the heat equation



on the Unit disk  $D = \{(r, \theta) \mid \partial \leq r \leq 1, \partial \leq \theta < 2\pi\}$ with  $U(1, \theta) = 1$ 

Suppose  $f(r,\Theta) = f(r)g(\Theta)$ for some real-valued functions f and g.  $\frac{\partial \upsilon}{\partial r} = f'(r)g(\Theta),$  $\frac{\partial^2 U}{\partial r^2} = f''(r)g(\Theta)_1$  $\partial \partial \partial \partial = f(r)g''(\partial)$ 

Plug into heat equation  $f''(r)g(\theta) + \frac{1}{r}f'(r)g(\theta) + \frac{1}{r^{2}}f(r)g''(\theta)$ Multiply through by ra.  $\Gamma^{2}(r)g(\Theta) + rf'(r)g(\Theta) + f(r)g''(\Theta) = 0$ 

Dividing by f(r)g(0), we get

 $\frac{f'(r) + rf'(r)}{f(r)} = -\frac{g''(\theta)}{g(\theta)}$ 

This means there must be a constant 2 such that  $\Gamma^{2}f''(r)+rf'(r) = 2 = -g''(\theta)$  f(r) f(r)

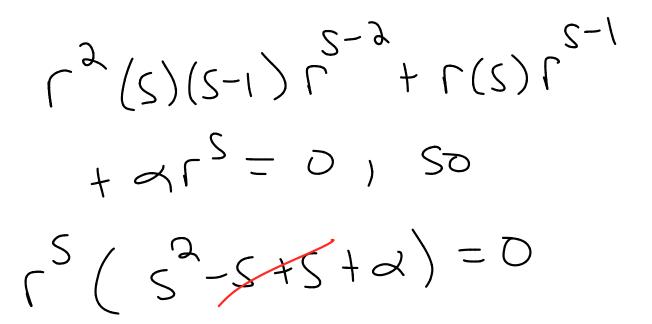
We get two differential equations

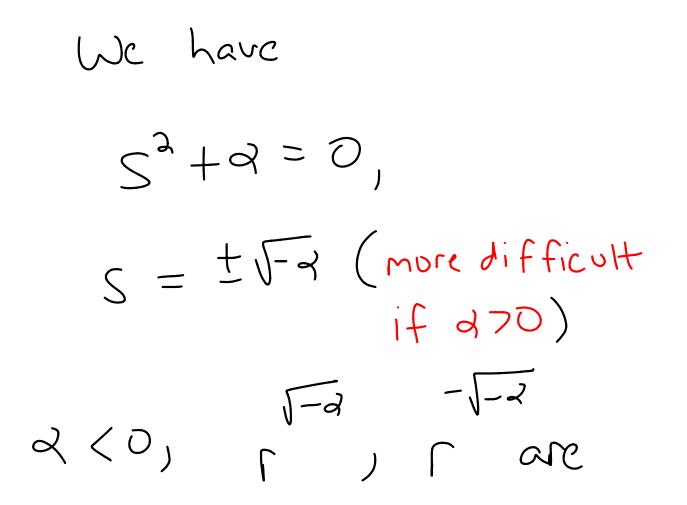
$$f''(r) + rf'(r) + \alpha f(r) = 0$$
  
 $g''(\theta) + \alpha g(\theta) = 0$ 

2 ra equation: Second order, Constant coefficients, homogeneous, so one solution is  $g(\Theta) = C^{\alpha \Theta}$ 

St Equation 
$$2^{nd}$$
 order, homogeneous,  
nonconstant coefficients - it is  
a Cauchy-Euler equation!  
One solution given by  
 $f(r) = r^{s}$ 

Solve:





two linearly independent solutions,

50

$$f(r) = C_1 \Gamma + C_2 \Gamma$$

More on Electrical Circuits (Section 5,7) Mea Culpa on Capacitors In a simple RC circuit,  $F(t) = E_t + E_R$ where  $E_R = RI$ ,  $E_c = \frac{4}{C}$ .

9 is nonconstant!

 $\frac{dq}{dt} = I(t), \quad so$ 

We way rewrite

 $R \frac{dq}{dt} + \frac{q}{c} = E(t)$